Inter (Part-I) 2019

Mathematics	Group-I	PAPER: I
Time: 2.30 Hours	(SUBJECTIVE TYPE)	Marks: 80

SECTION-I

2. Write short answers to any EIGHT (8) questions: (16)

(i) If z_1 and z_2 are complex numbers then show that $\overline{z_1 + z_2} = \overline{z_1} + \overline{z_2}$.

Ans Let $z_1 = a + ib$ and $z_2 = c + id$, then

$$z_1 + z_2 = (a + ib) + (c + id)$$
$$= (a + c) + i(b + d)$$

so, $\overline{z_1 + z_2} = \overline{(a + b) + i(b + d)}$

(Taking conjugate on both sides)

$$= (a + c) - i(b + d)$$
$$= (a - ib) + (c - id) = \overline{z_1} + \overline{z_2}$$

(ii) Find out real and imaginary parts of $(\sqrt{3} + i)^3$.

Ans Let $r\cos\theta = \sqrt{3}$ and $r\sin\theta = 1$ where

$r^2 = (\sqrt{3})^2 + 1^2$ or $r = \sqrt{3+1} = 2$ and $\theta = \tan^{-1}\dfrac{1}{\sqrt{3}} = 30^\circ$

So, $(\sqrt{3} + i)^3 = (r\cos\theta + i\, r\sin\theta)^3$

$$= r^3(\cos 3\theta + i\sin 3\theta) \text{ (By De Moivre's Theorem)}$$
$$= 2^3(\cos 90^\circ + i\sin 90^\circ)$$
$$= 8(0 + i\,.\,1) = 8i$$

Thus, 0 and 8 are respectively real and imaginary parts of $(\sqrt{3} + i)^3$.

(iii) Factorize $a^2 + 4b^2$.

Ans

$$a^2 + 4b^2 = a^2 - (2ib)^2$$
$$= (a)^2 - (2ib)^2 = (a - 2ib)(a + 2ib)$$

(iv) Define power set of a set and give an example.

Ans A set may contain elements, which are sets themselves. For example, if: C = Set of classes of a certain school, then elements of C are sets themselves because each class is a set of students. An important set of sets is the power set of a given set.

The power set of a set S denoted by P(S) is the set containing all the possible subsets of S.

(v) Define a bijective function.

Ans If f is a function from A onto B such that second elements of no two of its ordered pairs are the same, then f is said to be (1 – 1) function from A onto B. Such a function is also called a (1 – 1) correspondence between A and B. It is also called a bijective function.

(vi) Construct truth table and show that the statement $\sim(p \rightarrow q) \rightarrow p$ is a tautology or not.

Ans

p	q	$p \rightarrow q$	$\sim(p \rightarrow q)$	$\sim(p \rightarrow q) \rightarrow p$
T	T	T	F	T
T	F	F	T	T
F	T	T	F	T
F	F	T	F	T

Since all the possible values of $\sim(p \rightarrow q) \rightarrow p$ are true. Thus $\sim(p \rightarrow q) \rightarrow p$ is a tautology.

(vii) Find the matrix X if $X\begin{bmatrix} 5 & 2 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 12 & 3 \end{bmatrix}$.

Ans
$$X\begin{bmatrix} 5 & 2 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 12 & 3 \end{bmatrix} \quad (1)$$

Let, $X = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

then (i) $\Rightarrow \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 5 & 2 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 12 & 3 \end{bmatrix}$

Comparing the corresponding elements,

$$\begin{bmatrix} 5a - 2b & 2a + b \\ 5c - 2d & 2c + d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 12 & 3 \end{bmatrix}$$

Comparing corresponding elements,

$$5a - 2b = -1 \quad (i)$$
$$2a + b = 5 \quad (ii)$$
$$5c - 2d = 12 \quad (iii)$$
$$2c + d = 3 \quad (iv)$$

Multiplying eq. (ii) by 2, then adding in eq. (i)

$$4a + 2b = 10$$
$$5a - 2b = -1$$

$$9a = 9 \quad \Rightarrow \quad \boxed{a = 1}$$

Put $a = 1$ in eq. (i),

$$5a - 2b = -1$$
$$5(1) - 2b = -1$$
$$-2b = -6$$
$$b = 3 \quad \Rightarrow \quad \boxed{b = 3}$$

Multiplying eq. (iv) by 2, then adding in eq. (iii)

$$4c + 2d = 6$$
$$5c - 2d = 12$$
$$\overline{9c = 18}$$
$$c = 2 \quad \Rightarrow \quad \boxed{c = 2}$$

Put $c = 2$ in eq. (iii),

$$5c - 2d = 12$$
$$5(2) - 2d = 12$$
$$10 - 2d = 12$$
$$-2d = 2 \quad \Rightarrow \quad \boxed{d = -1}$$

Hence the required matrix,

$$X = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}$$

(viii) For the matrix $A = \begin{bmatrix} 1 & -2 & 3 \\ -2 & 3 & 1 \\ 4 & -3 & 2 \end{bmatrix}$, find cofactor A_{12}.

Ans $M_{12} = \begin{vmatrix} -2 & 1 \\ 4 & 2 \end{vmatrix} = -4 - 4 = -8;$

Thus $A_{12} = (-1)^{1+2} M_{12} = (-1)^3(-8) = (-1)(-8) = 8.$

(ix) Without expansion show that $\begin{vmatrix} \alpha & \beta+\gamma & 1 \\ \beta & \gamma+\alpha & 1 \\ \gamma & \alpha+\beta & 1 \end{vmatrix} = 0.$

Ans $\text{L.H.S} = \begin{vmatrix} \alpha & \beta+\gamma & 1 \\ \beta & \gamma+\alpha & 1 \\ \gamma & \alpha+\beta & 1 \end{vmatrix}$

By $(c_1 + c_2)$,

$$= \begin{vmatrix} \alpha+\beta+\gamma & \beta+\gamma & 1 \\ \alpha+\beta+\gamma & \gamma+\alpha & 1 \\ \alpha+\beta+\gamma & \alpha+\beta & 1 \end{vmatrix}$$

Taking common $(\alpha + \beta + \gamma)$ from c_1,

$$= (\alpha + \beta + \gamma)\begin{vmatrix} 1 & \beta+\gamma & 1 \\ 1 & \gamma+\alpha & 1 \\ 1 & \alpha+\beta & 1 \end{vmatrix}.$$

$$= (\alpha + \beta + \gamma)(0) = 0 = \text{R.H.S.}$$

(x) When $x^4 + 2x^3 + kx^2 + 3$ is divided by $(x - 2)$, the remainder is 1. Find the value of k.

Ans Here, $P(x) = x^4 + 2x^3 + kx^2 + 3$ and $x - r = x - 2 \Rightarrow r = 2$

By Remainder Theorem, we have

Remainder $1 = R = P(r) = P(2) = (2)^4 + 2(2)^3 + k(2)^2 + 3$

$$= 16 + 16 + 4k + 3 = 35 + 4k$$

$$\therefore\ 1 = 35 + 4k \Rightarrow 4k = 1 - 35 = -34 \Rightarrow k = -\frac{34}{4} = -\frac{17}{2}$$

(xi) If α, β are the roots of $ax^2 + bx + c = 0$, $a \neq 0$, then find the value of $\alpha^2 + \beta^2$.

Ans $ax^2 + bx + c = 0$

If α, β are the roots of the above equation,

$$\alpha + \beta = -\frac{b}{a}$$

$$\alpha\beta = \frac{c}{a}$$

Now $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$$= \left(-\frac{b}{a}\right)^2 - 2\left(\frac{c}{a}\right) = \frac{b^2}{a^2} - \frac{2c}{a} = \frac{b^2 - 2ac}{a^2}$$

(xii) The sum of a positive number and its square is 380. Find the number.

Ans Let the required number = x

According to given condition, we have $x + x^2 = 380$

$\Rightarrow\quad x^2 + x - 380 = 0$

$\Rightarrow\quad x^2 - 19x + 20x - 380 = 0$

$\Rightarrow\quad x(x - 19) + 20(x - 19) = 0$

$\Rightarrow\quad (x - 19)(3x + 20) = 0$

$\Rightarrow\quad x = 19, -20$

$\therefore$ $\boxed{\text{Required positive method} = 19}$

3. Write short answers to any EIGHT (8) questions: (16)

(i) Define partial fraction.

Ans If a fraction can be written as the sum of separate fractions, then these separate fractions are called the partial fractions of the original fraction. For example, the fraction $\frac{3}{(x+1)(x-1)}$ can be written as a sum of two separate fractions $= \frac{3}{2(x+1)}$ and $\frac{3}{2(x-1)}$ that is

$$\frac{3}{(x+1)(x-1)} = -\frac{3}{2(x+1)} + \frac{3}{2(x-1)}$$

(ii) In the identity $7x + 25 = A(x + 4) + B(x + 3)$, calculate values of A and B.

Ans Suppose $\frac{7x+25}{(x+3)(x+4)} = \frac{A}{x+3} + \frac{B}{x+4}$

$\Rightarrow \quad 7x + 25 = A(x + 4) + B(x + 3)$

As two sides of the identity are equal for all values of x, Let us put $x = -3$, and $x = -4$ in it.

$$7(-3) + 25 = A(-3 + 4) + B(-3 + 3)$$

Putting $\quad x = -3,$

we get $\quad -21 + 25 = A(-3 + 4) \qquad \boxed{A = 4}$

Putting $\quad x = -4$

we get $\quad -28 + 25 = -B(-4 + 3) \qquad \boxed{B = 3}$

Hence the partial fractions are:

$$\frac{4}{x+3} + \frac{3}{x+4}$$

(iii) Resolve $\frac{1}{x^2-1}$ into partial fractions.

Ans

$$\frac{1}{x^2-1} = \frac{1}{(x+1)(x-1)}$$

$$\frac{1}{(x+1)(x-1)} = \frac{A}{x+1} + \frac{B}{x-1}$$

$$1 = A(x-1) + B(x+1) \qquad (1)$$

Put $\quad x + 1 = 0$

$1 = -2A$

$A = -\frac{1}{2}$

Now put $x - 1 = 0$

$x = 1$ in (1), we get.

$$1 = 2B$$

$$B = \frac{1}{2}$$

Now, $\frac{1}{(x+1)(x-1)} = \frac{-\frac{1}{2}}{x+1} + \frac{\frac{1}{2}}{x-1}$

$$= -\frac{1}{2(x+1)} + \frac{1}{2(x-1)}$$

Which are required partial fractions.

(iv) Write the first four terms of the sequence, if $a_n - a_{n-1} = n + 2$, $a_1 = 2$.

Ans Given that $a_n - a_{n-1} = n + 2$ and $a_1 = 2$

For getting required terms, we put

$n = 2, 3$ and 4

For $n = 2$, $a_2 - a_1 = 2 + 2$

$\Rightarrow$ $a_2 - 2 = 4 \Rightarrow a_2 = 6$

For $n = 3$, $a_3 - a_2 = 3 + 2$

$\Rightarrow$ $a_3 - 6 = 5 \Rightarrow a_3 = 11$

For $n = 4$, $a_4 - a_3 = 4 + 2$

$\Rightarrow$ $a_4 - 11 = 6 \Rightarrow a_4 = 17$

Thus the first four terms of the sequence are 2, 6, 11, 17.

(v) Which term of the arithmetic sequence 5, 2, –1, ---- is –85.

Ans Given, AP . 5, 2, –1, - - - , –85

Here $a = 5$,

$d = 2 - 5 = -3$

$a_n = -85$

$n = ?$

$$a_n = a + (n-1)d$$
$$-85 = 5 + (n-1)(-3)$$
$$-85 = 5 - 3n + 3$$
$$-85 = 8 - 3n$$
$$3n = 8 + 85$$
$$3n = 93$$
$$n = \frac{93}{3}$$
$$n = 31$$

Thus $a_{31} = -85$

(vi) Find three A.Ms between 3 and 11.

Ans Let A_1, A_2, A_3 be three A.M's between 3 and 11. Then $3, A_1, A_2, A_3, 11$ are in A.P.

Here, $a = 3, n = 5, a_5 = 11, d = ?$

Using $a_n = a + (n-1)d$

$a_5 = n + (5-1)d$

$\Rightarrow \quad 11 = 3 + 4d$

$\Rightarrow \quad 4d = 11 - 3$

$\Rightarrow \quad 4d = 8$

$\Rightarrow \quad d = 2$

Hence, $A_1 = a + d \quad = 3 + 2 = 5$

$A_2 = A_1 + d \quad = 5 + 2 = 7$

$A_3 = A_2 = d \quad = 7 + 2 = 9$

Thus three A.M's between 3 and 11 are 5, 7, 9.

(vii) If $\frac{1}{a}, \frac{1}{b}$ and $\frac{1}{c}$ are in G.P, show that common ratio is $\pm\sqrt{\frac{a}{c}}$.

Ans Given $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in G.P.

Let r be the common ratio of the G.P

$$\therefore \quad r = \frac{\frac{1}{b}}{\frac{1}{a}} = \frac{a}{b} \qquad \text{(i)}$$

$$\text{Also} \quad r = \frac{\frac{1}{c}}{\frac{1}{b}} = \frac{b}{c} \qquad \text{(ii)}$$

Multiply (i) and (ii),

$$r^2 = \frac{a}{b} \times \frac{b}{c}$$

$$r = \pm\sqrt{\frac{a}{c}}$$

(viii) Insert two G.Ms between 2 and 16.

Ans Let G_1, G_2 be the two G.Ms between 2 and 16.

Then $2, G_1, G_2, 16$ are in G.P.

Here, $a = 2, n = 4, a_4 = 16$

We know $a_n = a r^{n-1}$

For $n = 4$, $a_4 = a r^{4-1}$

$\Rightarrow \quad 16 = 2(r)^3$

$\Rightarrow \quad r^3 = \frac{16}{2} = 8$

$\Rightarrow \quad r^3 = (2)^3$

$\Rightarrow \quad r = 2$

Thus $G_1 = ar = 2(2) = 4$

$G_2 = ar^2 = 2(2)^2 = 8$

Thus the two G.M's between 2 and 16 are 4, 8.

(ix) Find the value of n when ${}^nC_{10} = \frac{12 \times 11}{2!}$.

Ans

$${}^nC_{10} = \frac{12 \times 11}{2!}$$

$$= \frac{12 \times 11 \times 10!}{2! \times 10!} = \frac{12!}{2!(12-2)!}$$

$${}^nC_{10} = {}^{12}C_2$$

$${}^nC_{n-10} = {}^{12}C_2$$

$$n = 12$$

(x) Show that $\frac{n^3 + 2n}{3}$ represents an integer for $n = 2, 3$.

Ans Let, $S(n) = \frac{n^3 + 2n}{3}$

1. When $n = 1$, $S(1)$ becomes

$$S(1) = \frac{1^3 + 2(1)}{3} = \frac{3}{3} = 1 \in Z$$

2. Let us assume that $S(n)$ is true for any $n = k \in W$, that is,

$$S(k) = \frac{k^3 + 2k}{3} \text{ represents an integer.}$$

Now we want to show that $S(k+1)$ is also an integer.
For $n = k + 1$, the statement becomes

$$S(k+1) = \frac{(k+1)^3 + 2(k+1)}{3}$$

$$= \frac{k^3 + 3k^2 + 3k + 1 + 2k + 2}{3} = \frac{(k^3 + 2k) + (3k^2 + 3k + 3)}{3}$$

$$= \frac{(k^3 + 2k) + 3(k^2 + k + 1)}{3} = \frac{k^3 + 2k}{3} + (k^2 + k + 1)$$

As $\frac{k^3 + 2k}{3}$ is an integer by assumption and we know that $(k^2 + k + 1)$ is an integer as $k \in W$.

S(k + 1) being sum of integers is an integer, thus the condition (2) is satisfied. Since both the conditions are satisfied, therefore, we conclude by mathematical induction that $\frac{n^3 + 2n}{3}$ represents an integer for all positive integral values of n.

(xi) Expand $\left(1 - \frac{3}{2}x\right)^{-2}$ up to 4 terms.

Ans $= \frac{1}{4}\left(1 - \frac{3}{2}x\right)^{-2}$

$$= \frac{1}{4}\left\{1 + (-2)\left(-\frac{3x}{2}\right) + \frac{(-2)(-2-1)}{2!}\left(-\frac{3x}{2}\right)^2 + \frac{(-2)(-2-1)(-2-2)}{3!}\left(-\frac{3x}{2}\right)^3 + \ldots\right\}$$

$$= \frac{1}{4}\left\{1 + 3x + \frac{27x^2}{4} + \frac{27x^3}{2} + \ldots\right\}$$

$$= \frac{1}{4} + \frac{3x}{4} + \frac{27x^2}{16} + \frac{27x^3}{8} + \ldots \text{ valid if } |x| < \frac{2}{3}$$

(xii) If x is so small that its square and higher power can be neglected, then show that $\frac{\sqrt{1+2x}}{\sqrt{1-x}} \approx 1 + \frac{3}{2}x$.

Ans L.H.S $= \frac{\sqrt{1+2x}}{\sqrt{1-x}} = (1 + 2x)^{1/2}(1 - x)^{-1/2}$ (i)

Take $(1 + 2x)^{1/2} = \left\{1 + \frac{1}{2}(2x) + \frac{\left(\frac{1}{2}\right)\left(\frac{1}{2} - 1\right)}{2!}(2x)^2 + \right\}$

$= \{1 + x\}$ Neglecting x^2 and higher powers of x

Now, $(1 - x)^{-1/2} = \left\{1 + \left(-\frac{1}{2}\right)(-x) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{1}{2} - 1\right)}{2!}(-x)^2 + \ldots\right\}$

$= \left\{1 + \frac{x}{2}\right\}$ neglecting x^2 and higher powers of x.

Putting in eq. (i), we get

L.H.S $= \{1 + x\}\left\{1 + \frac{x}{2}\right\}$

$$= 1 + \frac{x}{2} + x + \frac{x^2}{2} = 1 + \frac{x + 2x}{2} \qquad \text{neglecting } x^2$$

$$= 1 + \frac{3x}{2} \cong \text{R.H.S.}$$

4. Write short answers to any NINE (9) questions: (18)

(i) Find l, if $\theta = 65^\circ 20'$, r = 18 mm.

Ans Given, $r = 18$ mm

$$\pi = \frac{22}{7}$$

$$\theta = 65^\circ 20'$$

$$= \left(65 + \frac{20}{60}\right)^\circ = \left(65 + \frac{1}{3}\right)^\circ = \frac{196^\circ}{3}$$

$$\theta = \frac{196}{3} \times \frac{\pi}{180} \text{ radians}$$

$$= 1.1403 \text{ radians}$$

As $l = r\theta$

$$l = 18(1.1403)$$

$$l = 20.53 \text{ mm}$$

(ii) Prove $\sin^2\frac{\pi}{6} : \sin^2\frac{\pi}{4} : \sin^2\frac{\pi}{3} : \sin^2\frac{\pi}{2} = 1 : 2 : 3 : 4$.

Ans $$\text{L.H.S} = \sin^2\frac{\pi}{6} : \sin^2\frac{\pi}{4} : \sin^2\frac{\pi}{2} : \sin^2\frac{\pi}{2}$$

$$= \sin^2 30^\circ : \sin^2 45^\circ : \sin^2 60^\circ : \sin^2 90^\circ$$

$$= \left(\frac{1}{2}\right)^2 : \left(\frac{1}{\sqrt{2}}\right)^2 : \left(\frac{\sqrt{3}}{2}\right)^2 : (1)^2 = \frac{1}{4} : \frac{1}{2} : \frac{3}{4} : 1$$

Multiplying by 4,

$$= 1 : 2 : 3 : 4 = \text{R.H.S}$$

Hence $\sin^2\frac{\pi}{6} : \sin^2\frac{\pi}{4} : \sin^2\frac{\pi}{3} : \sin^2\frac{\pi}{2} = 1 : 2 : 3 : 4$.

(iii) Prove $\cos^2\theta - \sin^2\theta = \dfrac{1 - \tan^2\theta}{1 + \tan^2\theta}$.

Ans $$\text{R.H.S} = \frac{1 - \tan^2\theta}{1 + \tan^2\theta} = \frac{1 - \dfrac{\sin^2\theta}{\cos^2\theta}}{1 + \dfrac{\sin^2\theta}{\cos^2\theta}}$$

$$= 1 + \frac{x}{2} + x + \frac{x^2}{2} = 1 + \frac{x + 2x}{2} \qquad \text{neglecting } x^2$$

$$= 1 + \frac{3x}{2} \cong \text{R.H.S.}$$

4. Write short answers to any NINE (9) questions: (18)

(i) Find l, if $\theta = 65°20'$, r = 18 mm.

Ans Given, $r = 18$ mm

$$\pi = \frac{22}{7}$$

$$\theta = 65°20'$$

$$= \left(65 + \frac{20}{60}\right)^o = \left(65 + \frac{1}{3}\right)^o = \frac{196^o}{3}$$

$$\theta = \frac{196}{3} \times \frac{\pi}{180} \text{ radians}$$

$$= 1.1403 \text{ radians}$$

As $l = r\theta$

$$l = 18(1.1403)$$

$$l = 20.53 \text{ mm}$$

(ii) Prove $\sin^2\frac{\pi}{6} : \sin^2\frac{\pi}{4} : \sin^2\frac{\pi}{3} : \sin^2\frac{\pi}{2} = 1 : 2 : 3 : 4$.

Ans $$\text{L.H.S} = \sin^2\frac{\pi}{6} : \sin^2\frac{\pi}{4} : \sin^2\frac{\pi}{2} : \sin^2\frac{\pi}{2}$$

$$= \sin^2 30° : \sin^2 45° : \sin^2 60° : \sin^2 90°$$

$$= \left(\frac{1}{2}\right)^2 : \left(\frac{1}{\sqrt{2}}\right)^2 : \left(\frac{\sqrt{3}}{2}\right)^2 : (1)^2 = \frac{1}{4} : \frac{1}{2} : \frac{3}{4} : 1$$

Multiplying by 4,

$$= 1 : 2 : 3 : 4 = \text{R.H.S}$$

Hence $\sin^2\frac{\pi}{6} : \sin^2\frac{\pi}{4} : \sin^2\frac{\pi}{3} : \sin^2\frac{\pi}{2} = 1 : 2 : 3 : 4$.

(iii) Prove $\cos^2\theta - \sin^2\theta = \frac{1 - \tan^2\theta}{1 + \tan^2\theta}$.

Ans $$\text{R.H.S} = \frac{1 - \tan^2\theta}{1 + \tan^2\theta} = \frac{1 - \frac{\sin^2\theta}{\cos^2\theta}}{1 + \frac{\sin^2\theta}{\cos^2\theta}}$$

$$= \frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta + \sin^2\theta}$$

$$= \frac{\cos^2\theta - \sin^2\theta}{1} \qquad \therefore \quad \sin^2\theta + \cos^2\theta = 1$$

$$= \cos^2\theta - \sin^2\theta = \text{L.H.S}$$

(iv) Prove that $\tan 56^\circ = \dfrac{\cos 11^\circ + \sin 11^\circ}{\cos 11^\circ - \sin 11^\circ}$.

Ans Consider:

$$\text{R.H.S} = \tan 56^\circ = \tan(45^\circ + 11^\circ)$$

$$= \frac{\tan 45^\circ + \tan 11^\circ}{1 - \tan 45^\circ \tan 11^\circ} = \frac{1 + \tan 11^\circ}{1 - \tan 11^\circ}$$

$$= \frac{1 + \dfrac{\sin 11^\circ}{\cos 11^\circ}}{1 - \dfrac{\sin 11^\circ}{\cos 11^\circ}} = \frac{\cos 11^\circ + \sin 11^\circ}{\cos 11^\circ - \sin 11^\circ} = \text{L.H.S}$$

Hence, $\dfrac{\cos 11^\circ + \sin 11^\circ}{\cos 11^\circ - \sin 11^\circ} = \tan 56^\circ$

(v) Prove $\dfrac{1 - \cos\alpha}{\sin\alpha} = \tan\dfrac{\alpha}{2}$.

Ans $\text{L.H.S} = \dfrac{1 - \cos\alpha}{\sin\alpha} = \dfrac{2\sin^2\dfrac{\alpha}{2}}{2\sin\dfrac{\alpha}{2}\cos\dfrac{\alpha}{2}}$

$$= \frac{\sin\dfrac{\alpha}{2}}{\cos\dfrac{\alpha}{2}} = \tan\frac{\alpha}{2} = \text{R.H.S.}$$

(vi) Prove $\cos 20^\circ + \cos 100^\circ + \cos 140^\circ = 0$.

Ans $\text{L.H.S} = \cos 20^\circ + \cos 100^\circ + \cos 140^\circ$

$$= 2\cos\frac{20^\circ + 100^\circ}{2}\cos\frac{20^\circ - 100^\circ}{2} + \cos 140^\circ$$

$$= 2\cos 60^\circ \cos(-40^\circ) + \cos 140^\circ$$

$$= 2 \cdot \frac{1}{2}\cos 40^\circ + \cos 140^\circ = \cos 40^\circ + \cos 140^\circ$$

$$= 2\cos\frac{40^\circ + 140^\circ}{2}\cos\frac{40^\circ - 140^\circ}{2}$$

$$= 2\cos 90^\circ \cos(-50^\circ) = 0 = \text{R.H.S.}$$

Hence, $\cos 20^\circ + \cos 100^\circ + \cos 140^\circ = 0$

(vii) Find the period of $\tan\frac{x}{7}$.

Ans $\tan\frac{x}{7}$

$$= \tan\left(\frac{x}{7} + \pi\right)$$

$$= \tan . \frac{1}{7}(x + 7\pi)$$

Hence period of $\tan\frac{x}{7} = 7\pi$.

(viii) In ΔABC, $\beta = 60^\circ$, $\gamma = 15^\circ$, $b = \sqrt{6}$, find c.

Ans $\beta = 60^\circ$, $\gamma = 15^\circ$, $b = \sqrt{6}$

$c = ?$

$a = ?$

$\alpha = ?$

As $\alpha + \beta + \gamma = 180^\circ \Rightarrow \alpha + 60^\circ + 15^\circ = 180^\circ$

$\alpha = 180^\circ - 75^\circ = 105^\circ$

As $\frac{c}{\sin\gamma} = \frac{b}{\sin\beta}$ (law of sines)

$$c = \frac{b}{\sin\beta} . \sin\gamma$$

$$= \frac{\sqrt{6}}{\sin 60^\circ} . \sin 15^\circ$$

$$c = \frac{\sqrt{6}}{.866} \times 0.25882$$

$$c = 0.7320$$

Also $\frac{a}{\sin\alpha} = \frac{b}{\sin\beta}$

$$a = \frac{b}{\sin\beta} \times \sin\alpha$$

$$a = \frac{(\sqrt{6})}{\sin 60^\circ} \times \sin 105^\circ$$

$$= \frac{\sqrt{6} \times 0.9659}{0.866} = 2.732$$

$a = 2.732$, $c = 0.732$, $b = \sqrt{6}$, $\alpha = 105^\circ$

(ix) **If $a = 200$, $b = 120$, $\gamma = 150^\circ$, find the area of a triangle ABC.**

Ans $a = 200$, $b = 120$, $\gamma\ 150^\circ$

By area formula,

$$\text{Area of } \Delta = \frac{1}{2} ab \sin\gamma = \frac{1}{2}(200)(120)\sin 150^\circ$$

$$= \frac{200 \times 120 \times \sin 150^\circ}{2} = 6000 \text{ sq. units}$$

(x) **Prove that $r_1 r_2 r_3 = rs^2$.**

Ans L.H.S $\quad r_1 r_2 r_3 = \frac{\Delta}{s-a} \cdot \frac{\Delta}{s-b} \cdot \frac{\Delta}{s-c}$

$$= \frac{\Delta^3}{(s-a)(s-b)(s-c)} = \frac{s\Delta^3}{s(s-a)(s-b)(s-c)} = \frac{s\Delta^3}{\Delta^2}$$

$$= s\Delta = s\Delta \times \frac{s}{s} = s^2 \frac{\Delta}{s}$$

$$= s^2 r \qquad \because \text{As } \left(\frac{\Delta}{s} = r\right)$$

$= \text{R.H.S}$ $\qquad$ so proved.

(xi) **Prove $\sin(2\cos^{-1} x) = 2x\sqrt{1-x^2}$.**

Ans L.H.S $= \sin(2\cos^{-1} x)$

Let, $\quad \cos^{-1} x = \theta$

$\qquad x = \cos\theta$

Now,

L.H.S $= \sin 2\theta = 2\sin\theta.\cos\theta = 2\cos\theta.\sqrt{1-\cos^2\theta}$

$= 2x.\sqrt{1-x^2} = \text{R.H.S}$

(xii) **Solve $1 + \cos x = 0$.**

Ans $\quad 1 + \cos x = 0$

$\Rightarrow \quad \cos x = -1$

Since $\cos x$ is –ve, there is only one solution $x = \pi$ in $[0, 2\pi]$

Since 2π is the period of $\cos x$

$\therefore$ General value of x is $\pi + 2n\pi$, $\quad n \in Z$

Hence solution set $= \{\pi + 2n\pi\}$, $\quad n \in Z$

(xiii) **Find the solutions of $\sin x = -\frac{\sqrt{3}}{2}$ in $[0, 2\pi]$.**

Ans $\sin x = -\frac{\sqrt{3}}{2}$

$\therefore$ $\sin x$ is –ve in third and fourth quadrants with the angle $x = \frac{\pi}{3}$.

$$\therefore \quad x = \pi + \frac{\pi}{3} = \frac{4\pi}{3} \quad \text{and} \quad 2\pi - \frac{\pi}{3} = \frac{5\pi}{3} = \frac{4\pi}{3}, \frac{5\pi}{3}$$

SECTION-II

NOTE: Attempt any Three (3) questions.

Q.5.(a) Prove that all 2 × 2 non-singular matrices over the real field form a non-abelian group under multiplication. (5)

Ans Let M_2' represent the set of all 2 × 2 matrices of the form

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \text{ such that } |A| = a_{11}a_{22} - a_{12}a_{21} \neq 0.$$

For any $B \in M_2'$, we have

$$A = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

$$AB = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}\begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

$$= \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{21} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}$$

and $|AB| = (a_{11}b_{11} + a_{12}b_{21}).(a_{11}b_{21} + a_{12}b_{22}) - (a_{11}b_{12} + a_{12}b_{22})$
$(a_{21}b_{11} + a_{22}b_{21}) \neq 0$

Hence $AB \in M_2'$.

Thus the set of all 2 × 2 non-singular matrices over real field form a non-abelian group under multiplication.

(b) Find three, consecutive numbers in G.P whose sum is 26 and their product is 216. (5)

Ans Let the three consecutive numbers in geometric progression (G.P) be $\frac{a}{r}$, a, ar. Thus, by given conditions:

$$\frac{a}{r} + a + ar = 26$$

$$\frac{a + ar + ar^2}{r} = 26$$

$$a + ar + ar^2 = 26r \quad (1)$$

Again, $\left(\frac{a}{r}\right)(a)(ar) = 216$

$$a^3 = 216$$
$$(a^3)^{1/3} = (6^3)^{1/3}$$
$$a = 6$$

By putting a = 6 in equation (1), we get

$$6 + 6r + 6r^2 = 26r$$
$$6r^2 - 20r + 6 = 0$$
$$3r^2 - 10r + 3 = 0$$
$$3r^2 - 9r - r + 3 = 0$$
$$3r(r - 3) - 1(r - 3) = 0$$
$$(r - 3)(3r - 1) = 0$$
$$\Rightarrow \quad r = 3, \quad r = \frac{1}{3}$$

When $r = 3$

The three numbers in G.P are:

$$\frac{a}{r} = \frac{6}{3} = 2$$

Three numbers = (2, 6, 18)

and if $r = \frac{1}{3}$

The three numbers in G.P are:

$$\frac{a}{r} = \frac{6}{\frac{1}{3}} = 6(3) = 18$$
$$a = 6$$
$$ar = 6(3) = 18$$

The numbers = (18, 6, 2).

Q.6.(a) Find the inverse of the matrix $A = \begin{bmatrix} 2 & 5 & -1 \\ 3 & 4 & 2 \\ 1 & 2 & -2 \end{bmatrix}$ by using row operation. **(5)**

Ans $|A| = \begin{vmatrix} 2 & 5 & -1 \\ 3 & 4 & 2 \\ 1 & 2 & -2 \end{vmatrix} = 2(-8 - 4) - 5(-6 - 2) - 1(6 - 4)$

$$= -24 + 40 - 2 = 40 - 26 = 14$$

As $|A| \neq 0$, so A is non-singular.

Appending I_3 on the left of the matrix A, we have

$$\left[\begin{array}{ccc:ccc} 2 & 5 & -1 & 1 & 0 & 0 \\ 3 & 4 & 2 & 0 & 1 & 0 \\ 1 & 2 & -2 & 0 & 0 & 1 \end{array}\right]$$

Interchanging R_1 and R_3, we get

$$\left[\begin{array}{ccc:ccc} 1 & 2 & -2 & 0 & 0 & 1 \\ 3 & 4 & 2 & 0 & 1 & 0 \\ 2 & 5 & -1 & 1 & 0 & 0 \end{array}\right]$$

$$\underset{\sim}{R}\left[\begin{array}{ccc:ccc} 1 & 2 & -2 & 0 & 0 & 1 \\ 0 & -2 & 8 & 0 & 1 & -3 \\ 0 & 1 & 3 & 1 & 0 & -2 \end{array}\right] \begin{array}{l} \text{By } R_2 + (-3)R_1 \to R_2' \\ \text{and } R_3 + (-2)R_1 \to R_3' \end{array}$$

By $-\frac{1}{2}R_2 \to R_2'$, we get

$$\left[\begin{array}{ccc:ccc} 1 & 2 & -2 & 0 & 0 & 1 \\ 0 & 1 & -4 & 0 & -1/2 & 3/2 \\ 0 & 1 & 3 & 1 & 0 & -2 \end{array}\right] \underset{\sim}{R}$$

$$\left[\begin{array}{ccc:ccc} 1 & 0 & 6 & 0 & 1 & -2 \\ 0 & 1 & -4 & 0 & -1/2 & 3/2 \\ 0 & 0 & 7 & 1 & 1/2 & -7/2 \end{array}\right]$$

By $R_3 + (-1)R_2 \to R_3'$

and $R_1 + (-2)R_2 \to R_1'$

By $\frac{1}{7}R_3 \to R_3'$, we have

$$\left[\begin{array}{ccc:ccc} 1 & 0 & 6 & 0 & 1 & -2 \\ 0 & 1 & -4 & 0 & -1/2 & 3/2 \\ 0 & 0 & 1 & 1/7 & 1/14 & -1/2 \end{array}\right]$$

$$\underset{\sim}{R}\left[\begin{array}{ccc:ccc} 1 & 0 & 0 & -6/7 & 4/7 & 1 \\ 0 & 1 & 0 & 4/7 & -3/14 & -1/2 \\ 0 & 0 & 1 & 1/7 & 1/14 & -1/2 \end{array}\right] \begin{array}{l} \text{By } R_1 + (-6)R_3 \to R_1' \\ \text{and } R_2 + 4R_3 \to R_2' \end{array}$$

Thus the inverse of A is

$$\begin{bmatrix} -\frac{6}{7} & \frac{4}{7} & 1 \\ \frac{4}{7} & -\frac{3}{14} & -\frac{1}{2} \\ \frac{1}{7} & \frac{1}{14} & -\frac{1}{2} \end{bmatrix}$$

Appending I_3 below the matrices A, we have

$$\begin{bmatrix} 2 & 5 & -1 \\ 3 & 4 & 2 \\ 1 & 2 & -2 \\ \cdots & \cdots & \cdots \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Interchanging c_1 and c_3, we get

$$\begin{bmatrix} 2 & 5 & -1 \\ 3 & 4 & 2 \\ 1 & 2 & -2 \\ \cdots & \cdots & \cdots \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \underset{\sim}{C} \begin{bmatrix} -1 & 5 & 2 \\ 2 & 4 & 3 \\ -2 & 2 & 1 \\ \cdots & \cdots & \cdots \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \underset{\sim}{C} \begin{bmatrix} 1 & 5 & 2 \\ -2 & 4 & 3 \\ 2 & 2 & 1 \\ \cdots & \cdots & \cdots \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

By $(-1)\,C_1 \to C_1'$

By $C_2 + (-5)C_1 \to C_2'$ and $C_3 + (-2)C_1 \to C_3'$, we have

$$\begin{bmatrix} 1 & 0 & 0 \\ -2 & 14 & -7 \\ 2 & -8 & -3 \\ \cdots & \cdots & \cdots \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 5 & 2 \end{bmatrix} \underset{\sim}{C} \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 7 \\ 2 & -4/7 & -3 \\ \cdots & \cdots & \cdots \\ 0 & 0 & 1 \\ 0 & 1/14 & 0 \\ -1 & 5/14 & 2 \end{bmatrix} \text{ By } \frac{1}{14}C_2 \to C_2'$$

By $C_1 + (2)C_2 \to C_1'$ and $C_3 + (-7)\,C_2 \to C_3'$, we have

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \frac{6}{7} & -\frac{4}{7} & 1 \\ \cdots & \cdots & \cdots \\ 0 & 0 & 1 \\ \frac{1}{7} & \frac{1}{14} & -\frac{1}{2} \\ -\frac{2}{7} & \frac{5}{14} & -\frac{1}{2} \end{bmatrix} \underset{\sim}{C} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ \cdots & \cdots & \cdots \\ -\frac{6}{7} & \frac{4}{7} & 1 \\ \frac{4}{7} & -\frac{3}{14} & -\frac{1}{2} \\ \frac{1}{7} & \frac{1}{4} & -\frac{1}{2} \end{bmatrix}$$

By $C_1 + \left(-\frac{6}{7}\right) C_3 \to C_1'$

and $C_2 + \left(\frac{4}{7}\right) C_3 \to C_2'$

Thus the inverse of A is $\begin{bmatrix} -\frac{6}{7} & \frac{4}{7} & 1 \\ \frac{4}{7} & -\frac{3}{14} & -\frac{1}{2} \\ \frac{1}{7} & \frac{1}{14} & -\frac{1}{2} \end{bmatrix}$.

(b) Prove that ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$.

Ans L.H.S $= {}^nC_r + {}^nC_{r-1}$

$$= \frac{n!}{r!(n-r)!} + \frac{n!}{(r-1)!\,(n-r+1)!}$$

$$= \frac{n!}{r(r-1)!\,(n-r)!} + \frac{n!}{(r-1)!\,(n-r+1)(n-r)!}$$

$$= \frac{n!}{(r-1)!\,(n-r)!}\left\{\frac{1}{r} + \frac{1}{n-r+1}\right\}$$

$$= \frac{n!}{(r-1)!\,(n-r)!}\left\{\frac{n-r+1+r}{r(n-r+1)}\right\}$$

$$= \frac{(n+1)n!}{r(r-1)!\,(n-r+1)(n-r)!}$$

$$= \frac{(n+1)!}{r!\,(n-r+1)!}$$

$$= \frac{(n+1)!}{r!\,(n+1-r)!}$$

$$= {}^{n+1}C_r = \text{R.H.S}$$

Q.7.(a) Solve the system of equations: **(5)**

$$12x^2 - 25xy + 12y^2 = 0$$
$$4x^2 + 7y^2 = 148$$

Ans $12x^2 - 25xy + 12y^2 = 0 \Rightarrow 12x^2 - 16xy - 9xy + 12y^2 = 0,$

$\Rightarrow 4x(3x - 4y) - 3y(3x - 4y) = 0, \Rightarrow (3x - 4y)(4x - 3y) = 0$

(i) We solve $\boxed{3x - 4y = 0 \ldots \quad (1) \text{ and } 4x^2 + 7y^2 = 148 \ldots \quad (2)}$

Putting $x = \frac{4y}{3}$ from (1) in (2), we get

$$4\left(\frac{16y^2}{9}\right) + 7y^2 = 148 \Rightarrow 64y^2 + 63y^2 = 148 \times 9$$

$$\Rightarrow \quad 127y^2 = 148 \times 9 = 1332 \Rightarrow y = \pm\sqrt{\frac{1332}{127}}$$

$$\Rightarrow \quad 127x^2 = 148 \times 16 \Rightarrow y = \pm 3$$

When $y = \pm\sqrt{\frac{1332}{127}} \Rightarrow x = \frac{4y}{3} \Rightarrow x = \pm\frac{4}{3}\cdot\sqrt{\frac{1332}{127}}$

(ii) We solve $\boxed{4x - 3y = 0 \ldots \quad (3) \text{ and } 4x^2 + 7y^2 = 148 \ldots \quad (2)}$

Putting $y = \frac{4x}{3}$ from (3) in (2), we get

$$x^2 + y^2 = 45 \Rightarrow 4x^2 + 7 \cdot \frac{16x^2}{9} = 148 \Rightarrow 36x^2 + 112x^2 = 148 \times 9$$

$$\Rightarrow \quad 148x^2 = 148 \times 9 \Rightarrow x^2 = 9 \Rightarrow x = \pm 3$$

When $x = \pm 3 \Rightarrow y = \frac{4x}{3} \Rightarrow y = \pm 4 \Rightarrow \boxed{(3, 4), (-3, -4)}$

$$\therefore \quad \text{S.S.} = \left\{(3, 4), (-3, -4), \left(\pm\frac{4}{3}\cdot\sqrt{\frac{1332}{127}}, \pm\sqrt{\frac{1332}{127}}\right)\right\}$$

(b) If $y = \frac{1}{3} + \frac{1.3}{2!}\left(\frac{1}{3}\right)^2 + \frac{1.3.5}{3!}\left(\frac{1}{3}\right)^3 + \text{----}$ then prove that $y^2 + 2y - 2 = 0.$ **(5)**

Ans $y = \frac{1}{3} + \frac{1\,.\,3}{2!}\left(\frac{1}{3}\right)^2 + \frac{1\,.\,3\,.\,5}{3!}\left(\frac{1}{3}\right)^3 + \ldots \infty,$

Adding 1 on both sides, we get

$$1 + y = 1 + \frac{1}{3} + \frac{1\,.\,3}{2!}\left(\frac{1}{3}\right)^2 + \frac{1\,.\,3\,.\,5}{3!}\left(\frac{1}{3}\right)^3 + \ldots \infty \qquad \text{(i)}$$

The series is identical with the expansion of $(1 + x)^n$

i.e., $$(1 + x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots \qquad \text{(ii)}$$

Comparing second and third terms of (i) and (ii), we get

$$n\,x = \frac{1}{3} \qquad \text{(iii)}$$

and $$\frac{n(n-1)}{2!}x^2 = \frac{1\,.\,3}{2!}\left(\frac{1}{3}\right)^2$$

$$\Rightarrow \quad n(n-1)\,x^2 = \frac{1}{3} \qquad \text{(iv)}$$

Squaring eq. (iii), we get

$$n^2x^2 = \frac{1}{9}$$

Dividing (iv) by v, we get

$$\frac{n(n-1)x^2}{n^2x^2} = \frac{1}{3} \times 9$$

$$\frac{n-1}{n} = 3$$

$\Rightarrow \quad n - 1 = 3n \quad \Rightarrow \quad 3n - n = -1$

$\Rightarrow \quad 2n = -1 \quad \Rightarrow \quad n = -\frac{1}{2}$

Put $n = -\frac{1}{2}$ in equation (iii), we get

$$-\frac{1}{2}x = \frac{1}{3} \quad \Rightarrow \quad x = -\frac{2}{3}$$

Putting the values $x = -\frac{2}{3}$ and $n = -\frac{1}{2}$ in $1 + y = (1 + x)^n$

i.e., $\quad 1 + y = \left(1 - \frac{2}{3}\right)^{-½} = \left(\frac{1}{2}\right)^{-½}$

$\Rightarrow \quad 1 + y = 3^{½}$

Taking square on both sides, we get

$$(1 + y)^2 = 3$$

$\Rightarrow \quad y^2 + 2y + 1 = 3$

$\Rightarrow \quad y^2 + 2y - 2 = 0$

Q.8.(a) Prove that $\sqrt{\frac{1 - \sin\theta}{1 + \sin\theta}} = \sec\theta - \tan\theta$, where θ is not an odd multiple of $\frac{\pi}{2}$. (5)

Ans L.H.S. $= \sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = \sqrt{\frac{1-\sin\theta}{1+\sin\theta}}\sqrt{\frac{1-\sin\theta}{1-\sin\theta}}$ (rationalizing)

$$= \sqrt{\frac{(1-\sin\theta)^2}{1-\sin^2\theta}} = \sqrt{\frac{(1-\sin\theta)^2}{\cos^2\theta}} = \frac{1-\sin\theta}{\cos\theta}$$

$$= \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta} = \sec\theta - \tan\theta = \text{R.H.S.}$$

Hence, $\quad \sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = \sec\theta - \tan\theta.$

(b) If α, β, γ are the angles of a triangle ABC, then show that: (5)

$$\cot\frac{\alpha}{2} + \cot\frac{\beta}{2} + \cot\frac{\gamma}{2} = \cot\frac{\alpha}{2}\cot\frac{\beta}{2}\cot\frac{\gamma}{2}$$

Ans Since α, β, γ are the angles of a triangle

$$\therefore \quad \alpha + \beta + \gamma = 180^\circ$$

$$\Rightarrow \quad \alpha + \beta = 180^\circ - \gamma$$

$$\Rightarrow \quad \frac{\alpha}{2} + \frac{\beta}{2} = 90^\circ - \frac{\gamma}{2}$$

$$\Rightarrow \quad \tan\left(\frac{\alpha}{2} + \frac{\beta}{2}\right) = \tan\left(90^\circ - \frac{\gamma}{2}\right)$$

$$\Rightarrow \quad \frac{\tan\frac{\alpha}{2} + \tan\frac{\beta}{2}}{1 - \tan\frac{\alpha}{2}\tan\frac{\beta}{2}} = \cot\frac{\gamma}{2}$$

$$\Rightarrow \quad \frac{\frac{1}{\cot\frac{\alpha}{2}} + \frac{1}{\cot\frac{\beta}{2}}}{1 - \frac{1}{\cot\frac{\alpha}{2}}\frac{1}{\cot\frac{\beta}{2}}} = \cot\frac{\gamma}{2}$$

$$\Rightarrow \quad \frac{\dfrac{\cot\frac{\beta}{2} + \cot\frac{\alpha}{2}}{\cot\frac{\alpha}{2}\cot\frac{\beta}{2}}}{\dfrac{\cot\frac{\alpha}{2}\cot\frac{\beta}{2} - 1}{\cot\frac{\alpha}{2}\cot\frac{\beta}{2}}} = \cot\frac{\gamma}{2}$$

$$\Rightarrow \quad \frac{\cot\frac{\alpha}{2} + \cot\frac{\beta}{2}}{\cot\frac{\alpha}{2}\cot\frac{\beta}{2} - 1} = \cot\frac{\gamma}{2}$$

$$\Rightarrow \quad \cot\frac{\alpha}{2} + \cot\frac{\beta}{2} = \cot\frac{\alpha}{2}\cot\frac{\beta}{2}\cos\frac{\gamma}{2} - \cot\frac{\gamma}{2}$$

$$\Rightarrow \quad \cot\frac{\alpha}{2} + \cot\frac{\beta}{2} + \cot\frac{\gamma}{2} = \cot\frac{\alpha}{2}\cot\frac{\beta}{2}\cot\frac{\gamma}{2}$$

Q.9.(a) The sides of a triangle are $x^2 + x + 1$, $2x + 1$ and $x^2 - 1$. Prove that the greatest angle of the triangle is 120°. (5)

Ans For Answer see Paper 2018 (Group-II), Q.9.(a).

(b) Prove that $\tan^{-1}\frac{3}{4} + \tan^{-1}\frac{3}{5} - \tan^{-1}\frac{8}{19} = \frac{\pi}{4}$. (5)

Ans For Answer see Paper 2017 (Group-II), Q.9.(b).